Rules of indices

A LEVEL LINKS
Scheme of work: 1a. Algebraic expressions – basic algebraic manipulation, indices and surds

Key points

- $a^m \times a^n = a^{m+n}$
- $\dfrac{a^m}{a^n} = a^{m-n}$
- $(a^m)^n = a^{mn}$
- $a^0 = 1$
- $a^{\frac{1}{n}} = \sqrt[n]{a}$ i.e. the nth root of a
- $a^{\frac{m}{n}} = \sqrt[n]{a^m} = \left(\sqrt[n]{a}\right)^m$
- $a^{-m} = \dfrac{1}{a^m}$
- The square root of a number produces two solutions, e.g. $\sqrt{16} = \pm 4$.

Examples

Example 1 Evaluate 10^0

$10^0 = 1$	Any value raised to the power of zero is equal to 1

Example 2 Evaluate $9^{\frac{1}{2}}$

$9^{\frac{1}{2}} = \sqrt{9}$ $= 3$	Use the rule $a^{\frac{1}{n}} = \sqrt[n]{a}$

Example 3 Evaluate $27^{\frac{2}{3}}$

$27^{\frac{2}{3}} = \left(\sqrt[3]{27}\right)^2$ $= 3^2$ $= 9$	**1** Use the rule $a^{\frac{m}{n}} = \left(\sqrt[n]{a}\right)^m$ **2** Use $\sqrt[3]{27} = 3$

Example 4 Evaluate 4^{-2}

$4^{-2} = \frac{1}{4^2}$ $= \frac{1}{16}$	**1** Use the rule $a^{-m} = \frac{1}{a^m}$ **2** Use $4^2 = 16$

Example 5 Simplify $\frac{6x^5}{2x^2}$

$\frac{6x^5}{2x^2} = 3x^3$	$6 \div 2 = 3$ and use the rule $\frac{a^m}{a^n} = a^{m-n}$ to give $\frac{x^5}{x^2} = x^{5-2} = x^3$

Example 6 Simplify $\frac{x^3 \times x^5}{x^4}$

$\frac{x^3 \times x^5}{x^4} = \frac{x^{3+5}}{x^4} = \frac{x^8}{x^4}$ $= x^{8-4} = x^4$	**1** Use the rule $a^m \times a^n = a^{m+n}$ **2** Use the rule $\frac{a^m}{a^n} = a^{m-n}$

Example 7 Write $\frac{1}{3x}$ as a single power of x

$\frac{1}{3x} = \frac{1}{3}x^{-1}$	Use the rule $\frac{1}{a^m} = a^{-m}$, note that the fraction $\frac{1}{3}$ remains unchanged

Example 8 Write $\frac{4}{\sqrt{x}}$ as a single power of x

$\frac{4}{\sqrt{x}} = \frac{4}{x^{\frac{1}{2}}}$ $= 4x^{-\frac{1}{2}}$	**1** Use the rule $a^{\frac{1}{n}} = \sqrt[n]{a}$ **2** Use the rule $\frac{1}{a^m} = a^{-m}$

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Practice

5 Simplify.

a $\frac{3x^2 \times x^3}{2x^2}$

b $\frac{10x^5}{2x^2 \times x}$

c $\frac{3x \times 2x^3}{2x^3}$

d $\frac{7x^3y^2}{14x^5y}$

e $\frac{y^2}{y^{\frac{1}{2}} \times y}$

f $\frac{c^{\frac{1}{2}}}{c^2 \times c^{\frac{3}{2}}}$

g $\frac{\left(2x^2\right)^3}{4x^0}$

h $\frac{x^{\frac{1}{2}} \times x^{\frac{3}{2}}}{x^{-2} \times x^3}$

Watch out!

Remember that any value raised to the power of zero is 1. This is the rule $a^0 = 1$.

6 Evaluate.

a $4^{-\frac{1}{2}}$

b $27^{-\frac{2}{3}}$

c $9^{-\frac{1}{2}} \times 2^3$

d $16^{\frac{1}{4}} \times 2^{-3}$

e $\left(\frac{9}{16}\right)^{-\frac{1}{2}}$

f $\left(\frac{27}{64}\right)^{-\frac{2}{3}}$

7 Write the following as a single power of x.

a $\frac{1}{x}$

b $\frac{1}{x^7}$

c $\sqrt[4]{x}$

d $\sqrt[5]{x^2}$

e $\frac{1}{\sqrt[3]{x}}$

f $\frac{1}{\sqrt[3]{x^2}}$

8 Write the following without negative or fractional powers.

a x^{-3}

b x^0

c $x^{\frac{1}{5}}$

d $x^{\frac{2}{5}}$

e $x^{-\frac{1}{2}}$

f $x^{-\frac{3}{4}}$

9 Write the following in the form ax^n.

a $5\sqrt{x}$

b $\frac{2}{x^3}$

c $\frac{1}{3x^4}$

d $\frac{2}{\sqrt{x}}$

e $\frac{4}{\sqrt[3]{x}}$

f 3

10 Write as sums of powers of x.

a $\frac{x^5+1}{x^2}$

b $x^2\left(x+\frac{1}{x}\right)$

c $x^{-4}\left(x^2+\frac{1}{x^3}\right)$

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Surds and rationalising the denominator

A LEVEL LINKS
Scheme of work: 1a. Algebraic expressions – basic algebraic manipulation, indices and surds

Key points

- A surd is the square root of a number that is not a square number, for example $\sqrt{2}, \sqrt{3}, \sqrt{5}$, etc.
- Surds can be used to give the exact value for an answer.
- $\sqrt{ab} = \sqrt{a} \times \sqrt{b}$
- $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$
- To rationalise the denominator means to remove the surd from the denominator of a fraction.
- To rationalise $\frac{a}{\sqrt{b}}$ you multiply the numerator and denominator by the surd $\sqrt{b}$
- To rationalise $\frac{a}{b+\sqrt{c}}$ you multiply the numerator and denominator by $b-\sqrt{c}$

Examples

Example 1 Simplify $\sqrt{50}$

$\sqrt{50} = \sqrt{25 \times 2}$	**1** Choose two numbers that are factors of 50. One of the factors must be a square number
$= \sqrt{25} \times \sqrt{2}$	**2** Use the rule $\sqrt{ab} = \sqrt{a} \times \sqrt{b}$
$= 5 \times \sqrt{2}$	**3** Use $\sqrt{25} = 5$
$= 5\sqrt{2}$	

Example 2 Simplify $\sqrt{147} - 2\sqrt{12}$

$\sqrt{147} - 2\sqrt{12}$ $= \sqrt{49 \times 3} - 2\sqrt{4 \times 3}$	**1** Simplify $\sqrt{147}$ and $2\sqrt{12}$. Choose two numbers that are factors of 147 and two numbers that are factors of 12. One of each pair of factors must be a square number
$= \sqrt{49} \times \sqrt{3} - 2\sqrt{4} \times \sqrt{3}$	**2** Use the rule $\sqrt{ab} = \sqrt{a} \times \sqrt{b}$
$= 7 \times \sqrt{3} - 2 \times 2 \times \sqrt{3}$	**3** Use $\sqrt{49} = 7$ and $\sqrt{4} = 2$
$= 7\sqrt{3} - 4\sqrt{3}$ $= 3\sqrt{3}$	**4** Collect like terms

Example 3 Simplify $\left(\sqrt{7}+\sqrt{2}\right)\left(\sqrt{7}-\sqrt{2}\right)$

$\left(\sqrt{7}+\sqrt{2}\right)\left(\sqrt{7}-\sqrt{2}\right)$ $=\sqrt{49}-\sqrt{7}\sqrt{2}+\sqrt{2}\sqrt{7}-\sqrt{4}$ $=7-2$ $=5$	**1** Expand the brackets. A common mistake here is to write $\left(\sqrt{7}\right)^2=49$ **2** Collect like terms: $-\sqrt{7}\sqrt{2}+\sqrt{2}\sqrt{7}$ $=-\sqrt{7}\sqrt{2}+\sqrt{7}\sqrt{2}=0$

Example 4 Rationalise $\frac{1}{\sqrt{3}}$

$\frac{1}{\sqrt{3}}=\frac{1}{\sqrt{3}}\times\frac{\sqrt{3}}{\sqrt{3}}$ $=\frac{1\times\sqrt{3}}{\sqrt{9}}$ $=\frac{\sqrt{3}}{3}$	**1** Multiply the numerator and denominator by $\sqrt{3}$ **2** Use $\sqrt{9}=3$

Example 5 Rationalise and simplify $\frac{\sqrt{2}}{\sqrt{12}}$

$\frac{\sqrt{2}}{\sqrt{12}}=\frac{\sqrt{2}}{\sqrt{12}}\times\frac{\sqrt{12}}{\sqrt{12}}$ $=\frac{\sqrt{2}\times\sqrt{4\times3}}{12}$ $=\frac{2\sqrt{2}\sqrt{3}}{12}$ $=\frac{\sqrt{2}\sqrt{3}}{6}$	**1** Multiply the numerator and denominator by $\sqrt{12}$ **2** Simplify $\sqrt{12}$ in the numerator. Choose two numbers that are factors of 12. One of the factors must be a square number **3** Use the rule $\sqrt{ab}=\sqrt{a}\times\sqrt{b}$ **4** Use $\sqrt{4}=2$ **5** Simplify the fraction: $\frac{2}{12}$ simplifies to $\frac{1}{6}$

Example 6 Rationalise and simplify $\frac{3}{2+\sqrt{5}}$

$\frac{3}{2+\sqrt{5}} = \frac{3}{2+\sqrt{5}} \times \frac{2-\sqrt{5}}{2-\sqrt{5}}$	**1** Multiply the numerator and denominator by $2-\sqrt{5}$
$= \frac{3\left(2-\sqrt{5}\right)}{\left(2+\sqrt{5}\right)\left(2-\sqrt{5}\right)}$	**2** Expand the brackets
$= \frac{6-3\sqrt{5}}{4+2\sqrt{5}-2\sqrt{5}-5}$	**3** Simplify the fraction
$= \frac{6-3\sqrt{5}}{-1}$ $= 3\sqrt{5}-6$	**4** Divide the numerator by -1 Remember to change the sign of all terms when dividing by -1

Practice

1 Simplify.

a $\sqrt{45}$ **b** $\sqrt{125}$

c $\sqrt{48}$ **d** $\sqrt{175}$

e $\sqrt{300}$ **f** $\sqrt{28}$

g $\sqrt{72}$ **h** $\sqrt{162}$

> **Hint**
>
> One of the two numbers you choose at the start must be a square number.

2 Simplify.

a $\sqrt{72}+\sqrt{162}$ **b** $\sqrt{45}-2\sqrt{5}$

c $\sqrt{50}-\sqrt{8}$ **d** $\sqrt{75}-\sqrt{48}$

e $2\sqrt{28}+\sqrt{28}$ **f** $2\sqrt{12}-\sqrt{12}+\sqrt{27}$

> **Watch out!**
>
> Check you have chosen the highest square number at the start.

3 Expand and simplify.

a $(\sqrt{2}+\sqrt{3})(\sqrt{2}-\sqrt{3})$ **b** $(3+\sqrt{3})(5-\sqrt{12})$

c $(4-\sqrt{5})(\sqrt{45}+2)$ **d** $(5+\sqrt{2})(6-\sqrt{8})$

4 Rationalise and simplify, if possible.

a $\frac{1}{\sqrt{5}}$

b $\frac{1}{\sqrt{11}}$

c $\frac{2}{\sqrt{7}}$

d $\frac{2}{\sqrt{8}}$

e $\frac{2}{\sqrt{2}}$

f $\frac{5}{\sqrt{5}}$

g $\frac{\sqrt{8}}{\sqrt{24}}$

h $\frac{\sqrt{5}}{\sqrt{45}}$

5 Rationalise and simplify.

a $\frac{1}{3-\sqrt{5}}$

b $\frac{2}{4+\sqrt{3}}$

c $\frac{6}{5-\sqrt{2}}$

Extend

6 Expand and simplify $\left(\sqrt{x}+\sqrt{y}\right)\left(\sqrt{x}-\sqrt{y}\right)$

7 Rationalise and simplify, if possible.

a $\frac{1}{\sqrt{9}-\sqrt{8}}$

b $\frac{1}{\sqrt{x}-\sqrt{y}}$

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Factorising expressions

A LEVEL LINKS
Scheme of work: 1b. Quadratic functions – factorising, solving, graphs and the discriminants

Key points

- Factorising an expression is the opposite of expanding the brackets.
- A quadratic expression is in the form $ax^2 + bx + c$, where $a \neq 0$.
- To factorise a quadratic equation find two numbers whose sum is b and whose product is ac.
- An expression in the form $x^2 - y^2$ is called the difference of two squares. It factorises to $(x - y)(x + y)$.

Examples

Example 1 Factorise $15x^2y^3 + 9x^4y$

$15x^2y^3 + 9x^4y = 3x^2y(5y^2 + 3x^2)$	The highest common factor is $3x^2y$. So take $3x^2y$ outside the brackets and then divide each term by $3x^2y$ to find the terms in the brackets

Example 2 Factorise $4x^2 - 25y^2$

$4x^2 - 25y^2 = (2x + 5y)(2x - 5y)$	This is the difference of two squares as the two terms can be written as $(2x)^2$ and $(5y)^2$

Example 3 Factorise $x^2 + 3x - 10$

$b = 3$, $ac = -10$	**1** Work out the two factors of $ac = -10$ which add to give $b = 3$ (5 and −2)
So $x^2 + 3x - 10 = x^2 + 5x - 2x - 10$	**2** Rewrite the b term ($3x$) using these two factors
$= x(x + 5) - 2(x + 5)$	**3** Factorise the first two terms and the last two terms
$= (x + 5)(x - 2)$	**4** $(x + 5)$ is a factor of both terms

Example 4 Factorise $6x^2 - 11x - 10$

$b = -11$, $ac = -60$ So $6x^2 - 11x - 10 = 6x^2 - 15x + 4x - 10$ $= 3x(2x - 5) + 2(2x - 5)$ $= (2x - 5)(3x + 2)$	**1** Work out the two factors of $ac = -60$ which add to give $b = -11$ (-15 and 4) **2** Rewrite the b term ($-11x$) using these two factors **3** Factorise the first two terms and the last two terms **4** $(2x - 5)$ is a factor of both terms

Example 5 Simplify $\frac{x^2 - 4x - 21}{2x^2 + 9x + 9}$

$\frac{x^2 - 4x - 21}{2x^2 + 9x + 9}$	**1** Factorise the numerator and the denominator
For the numerator: $b = -4$, $ac = -21$	**2** Work out the two factors of $ac = -21$ which add to give $b = -4$ (-7 and 3)
So $x^2 - 4x - 21 = x^2 - 7x + 3x - 21$ $= x(x - 7) + 3(x - 7)$ $= (x - 7)(x + 3)$	**3** Rewrite the b term ($-4x$) using these two factors **4** Factorise the first two terms and the last two terms **5** $(x - 7)$ is a factor of both terms
For the denominator: $b = 9$, $ac = 18$	**6** Work out the two factors of $ac = 18$ which add to give $b = 9$ (6 and 3)
So $2x^2 + 9x + 9 = 2x^2 + 6x + 3x + 9$ $= 2x(x + 3) + 3(x + 3)$ $= (x + 3)(2x + 3)$	**7** Rewrite the b term ($9x$) using these two factors **8** Factorise the first two terms and the last two terms **9** $(x + 3)$ is a factor of both terms
So $\frac{x^2 - 4x - 21}{2x^2 + 9x + 9} = \frac{(x - 7)(x + 3)}{(x + 3)(2x + 3)}$ $= \frac{x - 7}{2x + 3}$	**10** $(x + 3)$ is a factor of both the numerator and denominator so cancels out as a value divided by itself is 1

Practice

> **Hint**
>
> Take the highest common factor outside the bracket.

1 Factorise.

a $6x^4y^3 - 10x^3y^4$ **b** $21a^3b^5 + 35a^5b^2$

c $25x^2y^2 - 10x^3y^2 + 15x^2y^3$

2 Factorise

a $x^2 + 7x + 12$ **b** $x^2 + 5x - 14$

c $x^2 - 11x + 30$ **d** $x^2 - 5x - 24$

e $x^2 - 7x - 18$ **f** $x^2 + x - 20$

g $x^2 - 3x - 40$ **h** $x^2 + 3x - 28$

3 Factorise

a $36x^2 - 49y^2$ **b** $4x^2 - 81y^2$

c $18a^2 - 200b^2c^2$

4 Factorise

a $2x^2 + x - 3$ **b** $6x^2 + 17x + 5$

c $2x^2 + 7x + 3$ **d** $9x^2 - 15x + 4$

e $10x^2 + 21x + 9$ **f** $12x^2 - 38x + 20$

5 Simplify the algebraic fractions.

a $\dfrac{2x^2 + 4x}{x^2 - x}$ **b** $\dfrac{x^2 + 3x}{x^2 + 2x - 3}$

c $\dfrac{x^2 - 2x - 8}{x^2 - 4x}$ **d** $\dfrac{x^2 - 5x}{x^2 - 25}$

e $\dfrac{x^2 - x - 12}{x^2 - 4x}$ **f** $\dfrac{2x^2 + 14x}{2x^2 + 4x - 70}$

6 Simplify

a $\dfrac{9x^2 - 16}{3x^2 + 17x - 28}$ **b** $\dfrac{2x^2 - 7x - 15}{3x^2 - 17x + 10}$

c $\dfrac{4 - 25x^2}{10x^2 - 11x - 6}$ **d** $\dfrac{6x^2 - x - 1}{2x^2 + 7x - 4}$

Extend

7 Simplify $\sqrt{x^2 + 10x + 25}$

8 Simplify $\dfrac{(x+2)^2 + 3(x+2)^2}{x^2 - 4}$

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Completing the square

A LEVEL LINKS
Scheme of work: 1b. Quadratic functions – factorising, solving, graphs and the discriminants

Key points

- Completing the square for a quadratic rearranges $ax^2 + bx + c$ into the form $p(x + q)^2 + r$
- If $a \neq 1$, then factorise using a as a common factor.

Examples

Example 1 Complete the square for the quadratic expression $x^2 + 6x - 2$

$x^2 + 6x - 2$	**1** Write $x^2 + bx + c$ in the form $\left(x+\frac{b}{2}\right)^2 - \left(\frac{b}{2}\right)^2 + c$
$= (x + 3)^2 - 9 - 2$	
$= (x + 3)^2 - 11$	**2** Simplify

Example 2 Write $2x^2 - 5x + 1$ in the form $p(x + q)^2 + r$

$2x^2 - 5x + 1$	**1** Before completing the square write $ax^2 + bx + c$ in the form $a\left(x^2 + \frac{b}{a}x\right) + c$
$= 2\left(x^2 - \frac{5}{2}x\right) + 1$	**2** Now complete the square by writing $x^2 - \frac{5}{2}x$ in the form $\left(x+\frac{b}{2}\right)^2 - \left(\frac{b}{2}\right)^2$
$= 2\left[\left(x - \frac{5}{4}\right)^2 - \left(\frac{5}{4}\right)^2\right] + 1$	
$= 2\left(x - \frac{5}{4}\right)^2 - \frac{25}{8} + 1$	**3** Expand the square brackets – don't forget to multiply $\left(\frac{5}{4}\right)^2$ by the factor of 2
$= 2\left(x - \frac{5}{4}\right)^2 - \frac{17}{8}$	**4** Simplify

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Practice

1 Write the following quadratic expressions in the form $(x + p)^2 + q$

a $x^2 + 4x + 3$

b $x^2 - 10x - 3$

c $x^2 - 8x$

d $x^2 + 6x$

e $x^2 - 2x + 7$

f $x^2 + 3x - 2$

2 Write the following quadratic expressions in the form $p(x + q)^2 + r$

a $2x^2 - 8x - 16$

b $4x^2 - 8x - 16$

c $3x^2 + 12x - 9$

d $2x^2 + 6x - 8$

3 Complete the square.

a $2x^2 + 3x + 6$

b $3x^2 - 2x$

c $5x^2 + 3x$

d $3x^2 + 5x + 3$

Extend

4 Write $(25x^2 + 30x + 12)$ in the form $(ax + b)^2 + c$.

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Solving quadratic equations by factorisation

A LEVEL LINKS
Scheme of work: 1b. Quadratic functions – factorising, solving, graphs and the discriminants

Key points

- A quadratic equation is an equation in the form $ax^2 + bx + c = 0$ where $a \neq 0$.
- To factorise a quadratic equation find two numbers whose sum is b and whose products is ac.
- When the product of two numbers is 0, then at least one of the numbers must be 0.
- If a quadratic can be solved it will have two solutions (these may be equal).

Examples

Example 1 Solve $5x^2 = 15x$

$5x^2 = 15x$ $5x^2 - 15x = 0$ $5x(x - 3) = 0$ So $5x = 0$ or $(x - 3) = 0$ Therefore $x = 0$ or $x = 3$	**1** Rearrange the equation so that all of the terms are on one side of the equation and it is equal to zero. Do not divide both sides by x as this would lose the solution $x = 0$. **2** Factorise the quadratic equation. $5x$ is a common factor. **3** When two values multiply to make zero, at least one of the values must be zero. **4** Solve these two equations.

Example 2 Solve $x^2 + 7x + 12 = 0$

$x^2 + 7x + 12 = 0$ $b = 7$, $ac = 12$ $x^2 + 4x + 3x + 12 = 0$ $x(x + 4) + 3(x + 4) = 0$ $(x + 4)(x + 3) = 0$ So $(x + 4) = 0$ or $(x + 3) = 0$ Therefore $x = -4$ or $x = -3$	**1** Factorise the quadratic equation. Work out the two factors of $ac = 12$ which add to give you $b = 7$. (4 and 3) **2** Rewrite the b term ($7x$) using these two factors. **3** Factorise the first two terms and the last two terms. **4** $(x + 4)$ is a factor of both terms. **5** When two values multiply to make zero, at least one of the values must be zero. **6** Solve these two equations.

Example 3 Solve $9x^2 - 16 = 0$

$9x^2 - 16 = 0$ $(3x + 4)(3x - 4) = 0$ So $(3x + 4) = 0$ or $(3x - 4) = 0$ $x = -\frac{4}{3}$ or $x = \frac{4}{3}$	**1** Factorise the quadratic equation. This is the difference of two squares as the two terms are $(3x)^2$ and $(4)^2$. **2** When two values multiply to make zero, at least one of the values must be zero. **3** Solve these two equations.

Example 4 Solve $2x^2 - 5x - 12 = 0$

$b = -5$, $ac = -24$ So $2x^2 - 8x + 3x - 12 = 0$ $2x(x - 4) + 3(x - 4) = 0$ $(x - 4)(2x + 3) = 0$ So $(x - 4) = 0$ or $(2x + 3) = 0$ $x = 4$ or $x = -\frac{3}{2}$	**1** Factorise the quadratic equation. Work out the two factors of $ac = -24$ which add to give you $b = -5$. (-8 and 3) **2** Rewrite the b term ($-5x$) using these two factors. **3** Factorise the first two terms and the last two terms. **4** $(x - 4)$ is a factor of both terms. **5** When two values multiply to make zero, at least one of the values must be zero. **6** Solve these two equations.

Practice

1 Solve

a $6x^2 + 4x = 0$ **b** $28x^2 - 21x = 0$

c $x^2 + 7x + 10 = 0$ **d** $x^2 - 5x + 6 = 0$

e $x^2 - 3x - 4 = 0$ **f** $x^2 + 3x - 10 = 0$

g $x^2 - 10x + 24 = 0$ **h** $x^2 - 36 = 0$

i $x^2 + 3x - 28 = 0$ **j** $x^2 - 6x + 9 = 0$

k $2x^2 - 7x - 4 = 0$ **l** $3x^2 - 13x - 10 = 0$

2 Solve

a $x^2 - 3x = 10$ **b** $x^2 - 3 = 2x$

c $x^2 + 5x = 24$ **d** $x^2 - 42 = x$

e $x(x + 2) = 2x + 25$ **f** $x^2 - 30 = 3x - 2$

g $x(3x + 1) = x^2 + 15$ **h** $3x(x - 1) = 2(x + 1)$

Hint

Get all terms onto one side of the equation.

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Solving quadratic equations by using the formula

A LEVEL LINKS
Scheme of work: 1b. Quadratic functions – factorising, solving, graphs and the discriminants

Key points

- Any quadratic equation of the form $ax^2 + bx + c = 0$ can be solved using the formula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
- If $b^2 - 4ac$ is negative then the quadratic equation does not have any real solutions.
- It is useful to write down the formula before substituting the values for a, b and c.

Examples

Example 7 Solve $x^2 + 6x + 4 = 0$. Give your solutions in surd form.

$a = 1, b = 6, c = 4$ $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$	**1** Identify a, b and c and write down the formula. Remember that $-b \pm \sqrt{b^2 - 4ac}$ is all over $2a$, not just part of it.
$x = \frac{-6 \pm \sqrt{6^2 - 4(1)(4)}}{2(1)}$	**2** Substitute $a = 1$, $b = 6$, $c = 4$ into the formula.
$x = \frac{-6 \pm \sqrt{20}}{2}$	**3** Simplify. The denominator is 2, but this is only because $a = 1$. The denominator will not always be 2.
$x = \frac{-6 \pm 2\sqrt{5}}{2}$	**4** Simplify $\sqrt{20}$. $\sqrt{20} = \sqrt{4 \times 5} = \sqrt{4} \times \sqrt{5} = 2\sqrt{5}$
$x = -3 \pm \sqrt{5}$	**5** Simplify by dividing numerator and denominator by 2.
So $x = -3 - \sqrt{5}$ or $x = \sqrt{5} - 3$	**6** Write down both the solutions.

Example 8 Solve $3x^2 - 7x - 2 = 0$. Give your solutions in surd form.

$a = 3, b = -7, c = -2$ $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$	**1** Identify a, b and c, making sure you get the signs right and write down the formula. Remember that $-b \pm \sqrt{b^2 - 4ac}$ is all over $2a$, not just part of it.
$x = \frac{-(-7) \pm \sqrt{(-7)^2 - 4(3)(-2)}}{2(3)}$	**2** Substitute $a = 3$, $b = -7$, $c = -2$ into the formula.
$x = \frac{7 \pm \sqrt{73}}{6}$	**3** Simplify. The denominator is 6 when $a = 3$. A common mistake is to always write a denominator of 2.
So $x = \frac{7 - \sqrt{73}}{6}$ or $x = \frac{7 + \sqrt{73}}{6}$	**4** Write down both the solutions.

Practice

5 Solve, giving your solutions in surd form.

a $3x^2 + 6x + 2 = 0$ **b** $2x^2 - 4x - 7 = 0$

6 Solve the equation $x^2 - 7x + 2 = 0$

Give your solutions in the form $\frac{a \pm \sqrt{b}}{c}$, where a, b and c are integers.

7 Solve $10x^2 + 3x + 3 = 5$

Give your solution in surd form.

> **Hint**
>
> Get all terms onto one side of the equation.

Extend

8 Choose an appropriate method to solve each quadratic equation, giving your answer in surd form when necessary.

a $4x(x - 1) = 3x - 2$

b $10 = (x + 1)^2$

c $x(3x - 1) = 10$

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Solving linear simultaneous equations using the elimination method

A LEVEL LINKS
Scheme of work: 1c. Equations – quadratic/linear simultaneous

Key points

- Two equations are simultaneous when they are both true at the same time.
- Solving simultaneous linear equations in two unknowns involves finding the value of each unknown which works for both equations.
- Make sure that the coefficient of one of the unknowns is the same in both equations.
- Eliminate this equal unknown by either subtracting or adding the two equations.

Examples

Example 1 Solve the simultaneous equations $3x + y = 5$ and $x + y = 1$

$3x + y = 5$ $-\quad x + y = 1$ $2x \quad = 4$ So $x = 2$	**1** Subtract the second equation from the first equation to eliminate the y term.
Using $x + y = 1$ $2 + y = 1$ So $y = -1$	**2** To find the value of y, substitute $x = 2$ into one of the original equations.
Check: equation 1: $3 \times 2 + (-1) = 5$ YES equation 2: $2 + (-1) = 1$ YES	**3** Substitute the values of x and y into both equations to check your answers.

Example 2 Solve $x + 2y = 13$ and $5x - 2y = 5$ simultaneously.

$x + 2y = 13$ $+\quad 5x - 2y = 5$ $6x \quad = 18$ So $x = 3$	**1** Add the two equations together to eliminate the y term.
Using $x + 2y = 13$ $3 + 2y = 13$ So $y = 5$	**2** To find the value of y, substitute $x = 3$ into one of the original equations.
Check: equation 1: $3 + 2 \times 5 = 13$ YES equation 2: $5 \times 3 - 2 \times 5 = 5$ YES	**3** Substitute the values of x and y into both equations to check your answers.

Example 3 Solve $2x + 3y = 2$ and $5x + 4y = 12$ simultaneously.

$(2x + 3y = 2) \times 4 \rightarrow \quad 8x + 12y = 8$ $(5x + 4y = 12) \times 3 \rightarrow \quad 15x + 12y = 36$ $7x = 28$ So $x = 4$	**1** Multiply the first equation by 4 and the second equation by 3 to make the coefficient of y the same for both equations. Then subtract the first equation from the second equation to eliminate the y term.
Using $2x + 3y = 2$ $2 \times 4 + 3y = 2$ So $y = -2$	**2** To find the value of y, substitute $x = 4$ into one of the original equations.
Check: equation 1: $2 \times 4 + 3 \times (-2) = 2$ YES equation 2: $5 \times 4 + 4 \times (-2) = 12$ YES	**3** Substitute the values of x and y into both equations to check your answers.

Practice

Solve these simultaneously equations.

1 $4x + y = 8$
$x + y = 5$

2 $3x + y = 7$
$3x + 2y = 5$

3 $4x + y = 3$
$3x - y = 11$

4 $3x + 4y = 7$
$x - 4y = 5$

5 $2x + y = 11$
$x - 3y = 9$

6 $2x + 3y = 11$
$3x + 2y = 4$

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Solving linear and quadratic simultaneous equations

A LEVEL LINKS
Scheme of work: 1c. Equations – quadratic/linear simultaneous

Key points

- Make one of the unknowns the subject of the linear equation (rearranging where necessary).
- Use the linear equation to substitute into the quadratic equation.
- There are usually two pairs of solutions.

Examples

Example 1 Solve the simultaneous equations $y = x + 1$ and $x^2 + y^2 = 13$

$x^2 + (x + 1)^2 = 13$	**1** Substitute $x + 1$ for y into the second equation.
$x^2 + x^2 + x + x + 1 = 13$ $2x^2 + 2x + 1 = 13$	**2** Expand the brackets and simplify.
$2x^2 + 2x - 12 = 0$ $(2x - 4)(x + 3) = 0$	**3** Factorise the quadratic equation.
So $x = 2$ or $x = -3$	**4** Work out the values of x.
Using $y = x + 1$ When $x = 2$, $y = 2 + 1 = 3$ When $x = -3$, $y = -3 + 1 = -2$	**5** To find the value of y, substitute both values of x into one of the original equations.
So the solutions are $x = 2,\ y = 3$ and $x = -3, y = -2$	
Check: equation 1: $3 = 2 + 1$ YES and $-2 = -3 + 1$ YES equation 2: $2^2 + 3^2 = 13$ YES and $(-3)^2 + (-2)^2 = 13$ YES	**6** Substitute both pairs of values of x and y into both equations to check your answers.

Example 2 Solve $2x + 3y = 5$ and $2y^2 + xy = 12$ simultaneously.

$x = \frac{5-3y}{2}$	**1** Rearrange the first equation.
$2y^2 + \left(\frac{5-3y}{2}\right)y = 12$	**2** Substitute $\frac{5-3y}{2}$ for x into the second equation. Notice how it is easier to substitute for x than for y.
$2y^2 + \frac{5y-3y^2}{2} = 12$ $4y^2 + 5y - 3y^2 = 24$ $y^2 + 5y - 24 = 0$	**3** Expand the brackets and simplify.
$(y + 8)(y - 3) = 0$	**4** Factorise the quadratic equation.
So $y = -8$ or $y = 3$	**5** Work out the values of y.
Using $2x + 3y = 5$ When $y = -8$, $2x + 3 \times (-8) = 5$, $x = 14.5$ When $y = 3$, $2x + 3 \times 3 = 5$, $x = -2$ So the solutions are $x = 14.5$, $y = -8$ and $x = -2$, $y = 3$	**6** To find the value of x, substitute both values of y into one of the original equations.
Check: equation 1: $2 \times 14.5 + 3 \times (-8) = 5$ YES and $2 \times (-2) + 3 \times 3 = 5$ YES equation 2: $2\times(-8)^2 + 14.5\times(-8) = 12$ YES and $2 \times (3)^2 + (-2) \times 3 = 12$ YES	**7** Substitute both pairs of values of x and y into both equations to check your answers.

Practice

Solve these simultaneous equations.

1 $y = 2x + 1$
$x^2 + y^2 = 10$

2 $y = 6 - x$
$x^2 + y^2 = 20$

3 $y = x - 3$
$x^2 + y^2 = 5$

4 $y = 9 - 2x$
$x^2 + y^2 = 17$

5 $y = 3x - 5$
$y = x^2 - 2x + 1$

6 $y = x - 5$
$y = x^2 - 5x - 12$

7 $y = x + 5$
$x^2 + y^2 = 25$

8 $y = 2x - 1$
$x^2 + xy = 24$

9 $y = 2x$
$y^2 - xy = 8$

10 $2x + y = 11$
$xy = 15$

Extend

11 $x - y = 1$
$x^2 + y^2 = 3$

12 $y - x = 2$
$x^2 + xy = 3$

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Linear inequalities

A LEVEL LINKS **Scheme of work:** 1d. Inequalities – linear and quadratic (including graphical solutions)

Key points

- Solving linear inequalities uses similar methods to those for solving linear equations.
- When you multiply or divide an inequality by a negative number you need to reverse the inequality sign, e.g. < becomes >.

Examples

Example 1 Solve $-8 \leq 4x < 16$

$-8 \leq 4x < 16$ $-2 \leq x < 4$	Divide all three terms by 4.

Example 2 Solve $4 \leq 5x < 10$

$4 \leq 5x < 10$ $\frac{4}{5} \leq x < 2$	Divide all three terms by 5.

Example 3 Solve $2x - 5 < 7$

$2x - 5 < 7$ $2x < 12$ $x < 6$	**1** Add 5 to both sides. **2** Divide both sides by 2.

Example 4 Solve $2 - 5x \geq -8$

$2 - 5x \geq -8$ $-5x \geq -10$ $x \leq 2$	**1** Subtract 2 from both sides. **2** Divide both sides by −5. Remember to reverse the inequality when dividing by a negative number.

Example 5 Solve $4(x - 2) > 3(9 - x)$

$4(x - 2) > 3(9 - x)$ $4x - 8 > 27 - 3x$ $7x - 8 > 27$ $7x > 35$ $x > 5$	**1** Expand the brackets. **2** Add $3x$ to both sides. **3** Add 8 to both sides. **4** Divide both sides by 7.

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Practice

1 Solve these inequalities.

a $4x > 16$ b $5x - 7 \leq 3$ c $1 \geq 3x + 4$

d $5 - 2x < 12$ e $\frac{x}{2} \geq 5$ f $8 < 3 - \frac{x}{3}$

2 Solve these inequalities.

a $\frac{x}{5} < -4$ b $10 \geq 2x + 3$ c $7 - 3x > -5$

3 Solve

a $2 - 4x \geq 18$ b $3 \leq 7x + 10 < 45$ c $6 - 2x \geq 4$

d $4x + 17 < 2 - x$ e $4 - 5x < -3x$ f $-4x \geq 24$

4 Solve these inequalities.

a $3t + 1 < t + 6$ b $2(3n - 1) \geq n + 5$

5 Solve.

a $3(2 - x) > 2(4 - x) + 4$ b $5(4 - x) > 3(5 - x) + 2$

Extend

6 Find the set of values of x for which $2x + 1 > 11$ and $4x - 2 > 16 - 2x$.

Quadratic inequalities

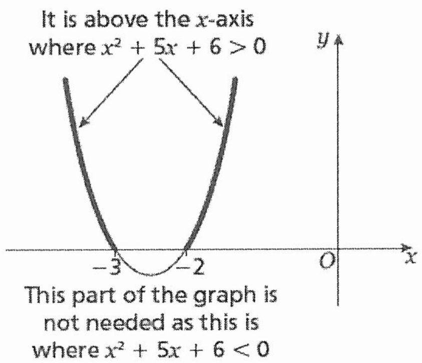
A LEVEL LINKS
Scheme of work: 1d. Inequalities – linear and quadratic (including graphical solutions)

Key points

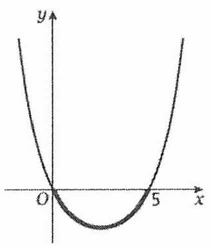
- First replace the inequality sign by = and solve the quadratic equation.
- Sketch the graph of the quadratic function.
- Use the graph to find the values which satisfy the quadratic inequality.

Examples

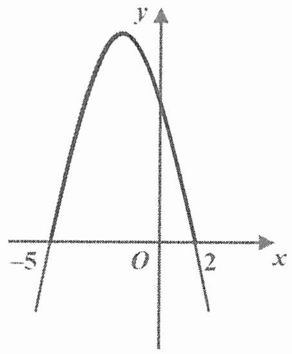
Example 1 Find the set of values of x which satisfy $x^2 + 5x + 6 > 0$

$x^2 + 5x + 6 = 0$ $(x + 3)(x + 2) = 0$ $x = -3$ or $x = -2$	**1** Solve the quadratic equation by factorising.
	**2** Sketch the graph of $y = (x + 3)(x + 2)$ **3** Identify on the graph where $x^2 + 5x + 6 > 0$, i.e. where $y > 0$
$x < -3$ or $x > -2$	**4** Write down the values which satisfy the inequality $x^2 + 5x + 6 > 0$

Example 2 Find the set of values of x which satisfy $x^2 - 5x \leq 0$

$x^2 - 5x = 0$ $x(x - 5) = 0$ $x = 0$ or $x = 5$	**1** Solve the quadratic equation by factorising.
	**2** Sketch the graph of $y = x(x - 5)$ **3** Identify on the graph where $x^2 - 5x \leq 0$, i.e. where $y \leq 0$
$0 \leq x \leq 5$	**4** Write down the values which satisfy the inequality $x^2 - 5x \leq 0$

Example 3 Find the set of values of x which satisfy $-x^2 - 3x + 10 \geq 0$

$-x^2 - 3x + 10 = 0$ $(-x + 2)(x + 5) = 0$ $x = 2$ or $x = -5$	**1** Solve the quadratic equation by factorising.
 	**2** Sketch the graph of $y = (-x + 2)(x + 5) = 0$ **3** Identify on the graph where $-x^2 - 3x + 10 \geq 0$, i.e. where $y \geq 0$
$-5 \leq x \leq 2$	**3** Write down the values which satisfy the inequality $-x^2 - 3x + 10 \geq 0$

Practice

1 Find the set of values of x for which $(x + 7)(x - 4) \leq 0$

2 Find the set of values of x for which $x^2 - 4x - 12 \geq 0$

3 Find the set of values of x for which $2x^2 - 7x + 3 < 0$

4 Find the set of values of x for which $4x^2 + 4x - 3 > 0$

5 Find the set of values of x for which $12 + x - x^2 \geq 0$

Extend

Find the set of values which satisfy the following inequalities.

6 $x^2 + x \leq 6$

7 $x(2x - 9) < -10$

8 $6x^2 \geq 15 + x$

Answers – Indices

5 **a** $\frac{3x^3}{2}$ **b** $5x^2$

c $3x$ **d** $\frac{y}{2x^2}$

e $y^{\frac{1}{2}}$ **f** c^{-3}

g $2x^6$ **h** x

6 **a** $\frac{1}{2}$ **b** $\frac{1}{9}$ **c** $\frac{8}{3}$

d $\frac{1}{4}$ **e** $\frac{4}{3}$ **f** $\frac{16}{9}$

7 **a** x^{-1} **b** x^{-7} **c** $x^{\frac{1}{4}}$

d $x^{\frac{2}{5}}$ **e** $x^{-\frac{1}{3}}$ **f** $x^{-\frac{2}{3}}$

8 **a** $\frac{1}{x^3}$ **b** 1 **c** $\sqrt[5]{x}$

d $\sqrt[5]{x^2}$ **e** $\frac{1}{\sqrt{x}}$ **f** $\frac{1}{\sqrt[4]{x^3}}$

9 **a** $5x^{\frac{1}{2}}$ **b** $2x^{-3}$ **c** $\frac{1}{3}x^{-4}$

d $2x^{-\frac{1}{2}}$ **e** $4x^{-\frac{1}{3}}$ **f** $3x^0$

10 **a** $x^3 + x^{-2}$ **b** $x^3 + x$ **c** $x^{-2} + x^{-7}$

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Answers – Surds

1 a $3\sqrt{5}$ b $5\sqrt{5}$
c $4\sqrt{3}$ d $5\sqrt{7}$
e $10\sqrt{3}$ f $2\sqrt{7}$
g $6\sqrt{2}$ h $9\sqrt{2}$

2 a $15\sqrt{2}$ b $\sqrt{5}$
c $3\sqrt{2}$ d $\sqrt{3}$
e $6\sqrt{7}$ f $5\sqrt{3}$

3 a -1 b $9-\sqrt{3}$
c $10\sqrt{5}-7$ d $26-4\sqrt{2}$

4 a $\frac{\sqrt{5}}{5}$ b $\frac{\sqrt{11}}{11}$
c $\frac{2\sqrt{7}}{7}$ d $\frac{\sqrt{2}}{2}$
e $\sqrt{2}$ f $\sqrt{5}$
g $\frac{\sqrt{3}}{3}$ h $\frac{1}{3}$

5 a $\frac{3+\sqrt{5}}{4}$ b $\frac{2(4-\sqrt{3})}{13}$ c $\frac{6(5+\sqrt{2})}{23}$

6 $x-y$

7 a $3+2\sqrt{2}$ b $\frac{\sqrt{x}+\sqrt{y}}{x-y}$

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Answers – factorising

1 a $2x^3y^3(3x - 5y)$ b $7a^3b^2(3b^3 + 5a^2)$
c $5x^2y^2(5 - 2x + 3y)$

2 a $(x + 3)(x + 4)$ b $(x + 7)(x - 2)$
c $(x - 5)(x - 6)$ d $(x - 8)(x + 3)$
e $(x - 9)(x + 2)$ f $(x + 5)(x - 4)$
g $(x - 8)(x + 5)$ h $(x + 7)(x - 4)$

3 a $(6x - 7y)(6x + 7y)$ b $(2x - 9y)(2x + 9y)$
c $2(3a - 10bc)(3a + 10bc)$

4 a $(x - 1)(2x + 3)$ b $(3x + 1)(2x + 5)$
c $(2x + 1)(x + 3)$ d $(3x - 1)(3x - 4)$
e $(5x + 3)(2x + 3)$ f $2(3x - 2)(2x - 5)$

5 a $\dfrac{2(x+2)}{x-1}$ b $\dfrac{x}{x-1}$
c $\dfrac{x+2}{x}$ d $\dfrac{x}{x+5}$
e $\dfrac{x+3}{x}$ f $\dfrac{x}{x-5}$

6 a $\dfrac{3x+4}{x+7}$ b $\dfrac{2x+3}{3x-2}$
c $\dfrac{2-5x}{2x-3}$ d $\dfrac{3x+1}{x+4}$

7 $(x + 5)$

8 $\dfrac{4(x+2)}{x-2}$

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Answers – completing the square

1 a $(x + 2)^2 - 1$ b $(x - 5)^2 - 28$

c $(x - 4)^2 - 16$ d $(x + 3)^2 - 9$

e $(x - 1)^2 + 6$ f $\left(x + \frac{3}{2}\right)^2 - \frac{17}{4}$

2 a $2(x - 2)^2 - 24$ b $4(x - 1)^2 - 20$

c $3(x + 2)^2 - 21$ d $2\left(x + \frac{3}{2}\right)^2 - \frac{25}{2}$

3 a $2\left(x + \frac{3}{4}\right)^2 + \frac{39}{8}$ b $3\left(x - \frac{1}{3}\right)^2 - \frac{1}{3}$

c $5\left(x + \frac{3}{10}\right)^2 - \frac{9}{20}$ d $3\left(x + \frac{5}{6}\right)^2 + \frac{11}{12}$

4 $(5x + 3)^2 + 3$

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Answers – solving quadratics

1 a $x = 0$ or $x = -\frac{2}{3}$

b $x = 0$ or $x = \frac{3}{4}$

c $x = -5$ or $x = -2$

d $x = 2$ or $x = 3$

e $x = -1$ or $x = 4$

f $x = -5$ or $x = 2$

g $x = 4$ or $x = 6$

h $x = -6$ or $x = 6$

i $x = -7$ or $x = 4$

j $x = 3$

k $x = -\frac{1}{2}$ or $x = 4$

l $x = -\frac{2}{3}$ or $x = 5$

2 a $x = -2$ or $x = 5$

b $x = -1$ or $x = 3$

c $x = -8$ or $x = 3$

d $x = -6$ or $x = 7$

e $x = -5$ or $x = 5$

f $x = -4$ or $x = 7$

g $x = -3$ or $x = 2\frac{1}{2}$

h $x = -\frac{1}{3}$ or $x = 2$

3 a $x = 2 + \sqrt{7}$ or $x = 2 - \sqrt{7}$

b $x = 5 + \sqrt{21}$ or $x = 5 - \sqrt{21}$

c $x = -4 + \sqrt{21}$ or $x = -4 - \sqrt{21}$

d $x = 1 + \sqrt{7}$ or $x = 1 - \sqrt{7}$

e $x = -2 + \sqrt{6.5}$ or $x = -2 - \sqrt{6.5}$

f $x = \frac{-3+\sqrt{89}}{10}$ or $x = \frac{-3-\sqrt{89}}{10}$

4 a $x = 1 + \sqrt{14}$ or $x = 1 - \sqrt{14}$

b $x = \frac{-3+\sqrt{23}}{2}$ or $x = \frac{-3-\sqrt{23}}{2}$

c $x = \frac{5+\sqrt{13}}{2}$ or $x = \frac{5-\sqrt{13}}{2}$

5 a $x = -1 + \frac{\sqrt{3}}{3}$ or $x = -1 - \frac{\sqrt{3}}{3}$

b $x = 1 + \frac{3\sqrt{2}}{2}$ or $x = 1 - \frac{3\sqrt{2}}{2}$

6 $x = \frac{7+\sqrt{41}}{2}$ or $x = \frac{7-\sqrt{41}}{2}$

7 $x = \frac{-3+\sqrt{89}}{20}$ or $x = \frac{-3-\sqrt{89}}{20}$

8 a $x = \frac{7+\sqrt{17}}{8}$ or $x = \frac{7-\sqrt{17}}{8}$

b $x = -1 + \sqrt{10}$ or $x = -1 - \sqrt{10}$

c $x = -1\frac{2}{3}$ or $x = 2$

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Answers – simultaneous equations

1 $x = 1, y = 4$

2 $x = 3, y = -2$

3 $x = 2, y = -5$

4 $x = 3, y = -\frac{1}{2}$

5 $x = 6, y = -1$

6 $x = -2, y = 5$

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Answers — linear & quadratic simultaneous equations

1 $x = 1, y = 3$

$x = -\frac{9}{5}, y = -\frac{13}{5}$

2 $x = 2, y = 4$

$x = 4, y = 2$

3 $x = 1, y = -2$

$x = 2, y = -1$

4 $x = 4, y = 1$

$x = \frac{16}{5}, y = \frac{13}{5}$

5 $x = 3, y = 4$

$x = 2, y = 1$

6 $x = 7, y = 2$

$x = -1, y = -6$

7 $x = 0, y = 5$

$x = -5, y = 0$

8 $x = -\frac{8}{3}, y = -\frac{19}{3}$

$x = 3, y = 5$

9 $x = -2, y = -4$

$x = 2, y = 4$

10 $x = \frac{5}{2}, y = 6$

$x = 3, y = 5$

11 $x = \frac{1+\sqrt{5}}{2}, y = \frac{-1+\sqrt{5}}{2}$

$x = \frac{1-\sqrt{5}}{2}, y = \frac{-1-\sqrt{5}}{2}$

12 $x = \frac{-1+\sqrt{7}}{2}, y = \frac{3+\sqrt{7}}{2}$

$x = \frac{-1-\sqrt{7}}{2}, y = \frac{3-\sqrt{7}}{2}$

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Answers – linear inequalities

1 a $x > 4$ b $x \leq 2$ c $x \leq -1$

d $x > -\frac{7}{2}$ e $x \geq 10$ f $x < -15$

2 a $x < -20$ b $x \leq 3.5$ c $x < 4$

3 a $x \leq -4$ b $-1 \leq x < 5$ c $x \leq 1$

d $x < -3$ e $x > 2$ f $x \leq -6$

4 a $t < \frac{5}{2}$ b $n \geq \frac{7}{5}$

5 a $x < -6$ b $x < \frac{3}{2}$

6 $x > 5$ (which also satisfies $x > 3$)

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Answers – quadratic inequalities

1 $-7 \leq x \leq 4$

2 $x \leq -2$ or $x \geq 6$

3 $\frac{1}{2} < x < 3$

4 $x < -\frac{3}{2}$ or $x > \frac{1}{2}$

5 $-3 \leq x \leq 4$

6 $-3 \leq x \leq 2$

7 $2 < x < 2\frac{1}{2}$

8 $x \leq -\frac{3}{2}$ or $x \geq \frac{5}{3}$